these theoretical curves is the same as those obtained from the experimental self-oscillating tunnel-diode mixer [1].

Conclusions

The following points are worthwhile considering:

1) The relatively small magnitudes of self-oscillations correspond to large gains. This is in agreement with the results obtained for externally applied local-oscillator tunnel-diode mixers [4].

2) In a tunnel-diode mixer with external local oscillator, both the bias voltage and the local oscillator magnitude can be varied independently. In the case of self-oscillating tunnel-diode mixer, the choice of bias voltage also fixes the oscillation magnitude and hence the infinite conversion loss condition becomes an inherent property of the mixer.

3) By suitable choice of G_L, the critical dependency of gain on the bias voltage can be minimized. However, in such an optimization procedure the noise figure of the mixer should also be considered.

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A New Method for Calculating the Capacitance of a Circular Disk for Microwave Integrated Circuits

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Abstract—**A method for calculating the capacitance of a circular disk on a dielectric substrate backed by a ground plane is presented. Hankel transforms and Galerkin's method are used to derive the expression for the capacitance. Numerical results are compared with the experimental data and good agreement is reported.**

The increasing use of integrated circuits (IC's) at microwave frequencies has created a great deal of interest in the theoretical and experimental studies of microstrip lines and other similar structures. However, most of these studies are concerned with the properties of infinitely long transmission lines [1]. In actual microwave IC's, many finite-sized or lumped elements are employed to realize the desired functional devices. Hence, the analysis of these finite-sized elements is also important; however, to date, very little has been reported on the analysis of such elements.

Among the finite elements, a rectangular microstrip was recently analyzed by Farrar and Adams [2] and Itoh *et al.* [3]. Another typical finite element is the circular disk (see Fig. 1) for which reliable design data are lacking. In the present short paper a new method is presented for calculating the total capacitance of the circular disk under the quasi-static approximation. The method is an extension of the spectral domain technique developed in [3].

The first step is to write Poisson's equation for the potential ϕ in the cylindrical coordinate

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}=-\frac{1}{\epsilon_0}\rho(r)\delta(z-d) \tag{1}$$

in which ρ is the charge distribution on the disk, and the θ terms vanished because of the circular symmetry. Let us now introduce the

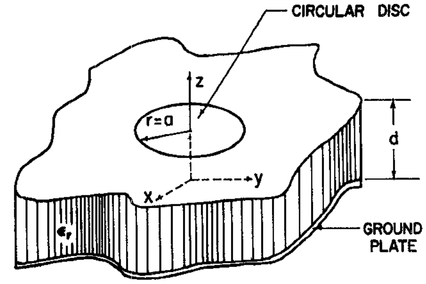

Fig. 1. Circular disk for the microwave integrated circuit.

Hankel transform of the order zero:

$$\tilde{\phi}(\alpha,z)=\int_0^\infty \phi(r,z)J_0(\alpha r)r\,dr. \tag{2}$$

Upon Hankel transforming (1) we obtain

$$\left(\frac{d^2}{dz^2}-\alpha^2\right)\tilde{\phi}(\alpha,z)=-\frac{1}{\epsilon_0}\tilde{\rho}(\alpha)\delta(z-d) \tag{3}$$

where

$$\tilde{\rho}(\alpha)=\int_0^a \rho(r)J_0(\alpha r)r\,dr$$

is the Hankel transform of the charge distribution. The general solution of (3) which satisfies the boundary condition $\tilde{\phi}(\alpha,0)=0$ and the radiation condition $\tilde{\phi}(\alpha,+\infty)=0$ is

$$\tilde{\phi}(\alpha,z)=\begin{cases}A(\alpha)\sinh\alpha z, & 0<z<d\\ B(\alpha)\exp\left[-\alpha(z-d)\right], & z>d.\end{cases} \tag{4}$$

The unknown coefficients $A(\alpha)$ and $B(\alpha)$ are determined so that the interface conditions

$$\tilde{\phi}(\alpha,d+0)=\tilde{\phi}(\alpha,d-0)$$

$$\frac{\partial}{\partial z}\tilde{\phi}(\alpha,d+0)-\epsilon_r\frac{\partial}{\partial z}\tilde{\phi}(\alpha,d-0)=-\frac{1}{\epsilon_0}\tilde{\rho}(\alpha)$$

are satisfied. Upon eliminating A and B we obtain

$$\tilde{G}(\alpha)\tilde{\rho}(\alpha)=\tilde{\phi}_i(\alpha,d)+\tilde{\phi}_0(\alpha,d) \tag{5}$$

where

$$\tilde{G}(\alpha)=\frac{1}{\epsilon_0\alpha[1+\epsilon_r\coth\alpha d]}$$

$$\tilde{\phi}_i(\alpha,d)=\int_0^a J_0(\alpha r)r\,dr=\frac{a}{\alpha}J_1(\alpha a)$$

$$\tilde{\phi}_0(\alpha,d)=\int_a^\infty \phi(r,d)J_0(\alpha r)r\,dr.$$

Equation (5) corresponds to the integral equation in the conventional space-domain formulation where the convolution integral appears instead of the product $\tilde{G}\tilde{\rho}$ found in (5). Note that (5) contains two unknowns $\tilde{\rho}$ and $\tilde{\phi}_0$. However, as will be shown shortly, $\tilde{\phi}_0$ is eliminated in the process of solution.

Galerkin's method is now applied to (5). As the first step toward this, $\tilde{\rho}(\alpha)$ is expanded in terms of the known basis functions $\tilde{\rho}_n(\alpha)$:

$$\tilde{\rho}(\alpha)=\sum_{n=1}^{N}d_n\tilde{\rho}_n(\alpha)$$

$$\tilde{\rho}_n(\alpha)=\int_0^a \rho_n(r)J_0(\alpha r)r\,dr \tag{6}$$

where $\rho_n(r)$, the inverse transforms of $\tilde{\rho}_n(\alpha)$, are chosen so that they are zero for $r>a$. Substituting (6) in (5) and taking an inner product of one of the $\tilde{\rho}_n$ with (5), we have

$$\sum_{n=1}^{N}K_{mn}d_n=a_m,\qquad m=1,2,\cdots,N \tag{7}$$

$$K_{mn}=\int_0^\infty \tilde{\rho}_m(\alpha)\tilde{G}(\alpha)\tilde{\rho}_n(\alpha)\alpha\,d\alpha \tag{8}$$

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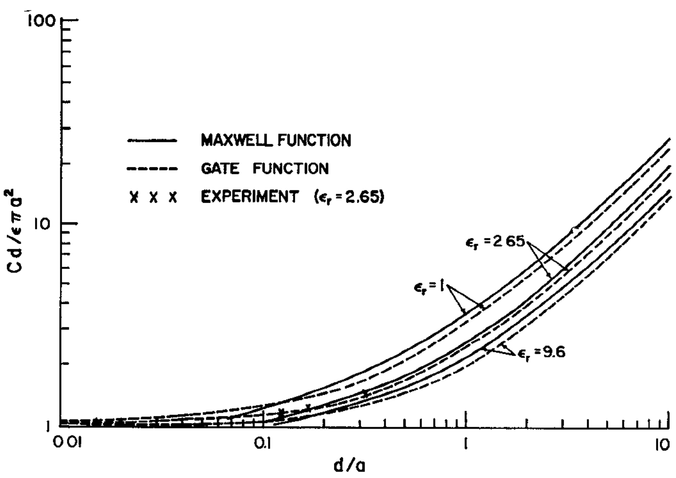

Fig. 2. Capacitance of a circular disk (normalized by $\epsilon\pi a^2/d$, $\epsilon = \epsilon_0\epsilon_r$).

$$a_m = \int_0^\infty \tilde{\rho}_m(\alpha)\tilde{\phi}_i(\alpha, d)\alpha \, d\alpha = \int_0^a \rho_m(r) r \, dr. \quad (9)$$

In the derivation of (9), Parseval's relation has been used; the use of this relation results in the elimination of $\tilde{\phi}_0$ since the inverses of $\tilde{\phi}_0$ and $\tilde{\rho}_m$ are nonzero only at the complementary regions of r.

The total capacitance of the structure is

$$C = \int_0^a 2\pi r\rho(r) \, dr = 2\pi \sum_{n=1}^{N} a_n d_n. \quad (10)$$

For the numerical calculation we have chosen $N=1$, although the accuracy of the result can be improved by increasing N. The two types of functions tested were as follows.

1) Maxwell function:

$$\rho_1(r) = \begin{cases} \dfrac{1}{\sqrt{a^2 - r^2}}, & r < a, \\ 0, & r > a. \end{cases} \qquad \tilde{\rho}_1(\alpha) = \frac{\sin \alpha a}{\alpha}$$

2) Gate function:

$$\rho_1(r) = \begin{cases} 1, & r < a, \\ 0, & r > a. \end{cases} \qquad \tilde{\rho}_1(\alpha) = \frac{aJ_1(\alpha a)}{\alpha}$$

Fig. 2 shows the capacitance of a circular disk for three different substrates, viz., $\epsilon_r = 1$, 2.65, and 9.6, calculated by using two different choices of basis functions. Note that there are crossover points for the two curves obtained by using the Maxwell function and the gate function. Since the capacitance given by (10) gives a stationary value for a trial set of basis functions, and since the one which maximizes the value of C yields a result closest to the exact one, it is evident that the Maxwell function should be used for d/a values above the crossover point, while the gate function will give more accurate results below it. The reason why the gate function gives better results for larger disks even though it ignores the edge behavior is perhaps due to the fact that the contribution of the edge singularity to the total capacitance of the large disk is a relatively small quantity. It is noted that for $d/a > 0.5$, the numerical results using the gate function are about 10 percent lower, and hence less accurate than the corresponding results for the Maxwell function.

The discrepancy between the two results is even greater for $d/a < 0.1$, where the gate function results are now more accurate.

As expected, for small values of d/a, C approaches $\epsilon_r\epsilon_0\pi a^2/d$, which is the value of the capacitance that would be obtained by neglecting the fringe effects. For $\epsilon_r = 1$, it is known that $Cd/(\epsilon\pi a^2)$ approaches $8d/(\pi a)$ as $d/a \to \infty$. For $d/a = 10$, this asymptotic value is approximately 25.5 and the numerical value computed by the present method is 26.2.

The required computation time for the above calculations with four-digit accuracy was about 6 s per structure for the choice 1) and 60 s for 2),[1] both on the CDC G-20 computer. For comparison purposes, this computer is about seven to ten times slower than the IBM 360/75.

In order to check the accuracy of the computed results, the capacitance of the actual circular disks on the substrate of $\epsilon_r = 2.65$ has been measured at 1.592 MHz. Fig. 2 shows that the experimental results are in excellent agreement with the numerical computation; in fact, the measured and the computed values differ by less than 3 percent. To conclude the discussion we might add that the principal advantage of the method is its numerical efficiency. An important feature of the method is that the numerical effort involved is not too dependent upon the physical size of the structure. In contrast, in most conventional numerical methods the computational effort is directly proportional to the size of the structure which in turn determines the size of the associated matrix.

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[1] This is because of the use of the time-consuming BESJ subroutine for the calculation of J_1. However, it is possible to reduce the computation time for J_1 by employing the polynomial approximations for J_1 given in [4].

Letters

Comments on "Analysis of Automatic Homodyne Method Amplitude and Phase Measurements"

GEORGE E. SCHAFER

In the above short paper,[1] on page 623, the authors state: "Phase quadrature between the homodyne and the modulated carriers produces a null in the detector output. . . ." This is only true if the modulated carrier is completely suppressed, which is the ideal case discussed by Robertson [9].[1] Inspection of the phasor when the carrier is not suppressed, as in Schafer [11],[1] shows that the null is produced when the modulated carrier is in phase quadrature with the resultant of the homodyne and modulated carriers. The error introduced by the authors' assumption of quadrature conditions varies from less than 0.01° for a 90-dB ratio to 90° for equality of the two signals. In most applications this error is less than 0.6° (40 dB or greater ratio), and for moderate accuracies it can be ignored. For more precise measurements, however, one must use the resultant and modulated carrier in phase-quadrature analysis.

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